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GROWTH LAW OF A FAST MOVING SPHERICAL SECOND PHASE AS GOVERNED BY SIMULTANEOUS HEAT AND MASS TRANSFER LIMITATIONS

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Abstract—New theoretical treatments of the growth of a fast moving spherical second phase as governed by simultaneous heat- and mass-transfer limitations are demonstrated. The new method demonstrates that the solution to these complex coupled cases can be related to the available uncoupled cases. Thus, the heat or mass transfer limited case is shown to be asymptotic cases of the simultaneous heat and mass transfer limited cases.

NOMENCLATURE

$A(t)$, function defined in equation (18);
$B(t)$, function defined in equation (19);
B_h^g, $\equiv c_p(T_\infty - T_w)/L(T_w)$ [dimensionless];
B_m^g, $\equiv (c_{sat}(T_w) - c_\infty)/(c_d - c_{sat}(T_w))$ [dimensionless];
c, mass fraction of solute [dimensionless];
c_p, specific heat of the first phase [cal/g°K];
D, effective Fick diffusion coefficient for solute transport in the first phase [cm²/s];
D/Dt, material time derivative defined in the text;
$f(t, \theta)$, function defined in equation (37b);
$G(t)$, function defined in equation (17);
$H(t)$, function defined in equations (37) and (38);
$L(T_w)$, latent heat of phase transition (<0 for endothermic; >0 for exothermic), [cal/g];
Le, $\equiv D/\alpha$, Lewis number [dimensionless];
Na_h^g $\equiv (\rho/\rho_d)\cdot B_h^g \equiv (\rho/\rho_d)\cdot[c_p(T_\infty - T_w)/L(T_w)]$ [dimensionless];
Na_m^g, $\equiv (\rho/\rho_d)\cdot B_m^g \equiv (\rho/\rho_d)\cdot[(c_{sat}(T_w) - c_\infty)/(c_d - c_{sat}(T_w))]$ [dimensionless];
p, dummy integration variable;
$R(t)$, instantaneous radius of the growing second phase sphere [cm];
$\dot{R}$, $\equiv dR/dt$ [cm/s];
R_0, initial radius of the growing second phase sphere [cm];
r, radial coordinate reckoned from center of second phase sphere [cm];
s, dummy integration variable;
t, time (reckoned from the commencement of growth process) [s];
T, temperature [°K];
T_w, sphere surface temperature [°K];
u_∞, translational velocity of the center of second phase sphere relative to the surrounding fluid at infinity [cm/s];
v_{rg}, radially spherically symmetric convective velocity field induced by mass transfer process itself [cm/s];
v_{rt}, radial velocity component field induced by the translatory motion of the second phase sphere [cm/s];
$v_{\theta t}$, tangential velocity component field induced by the translatory motion of

the second phase sphere [cm/s];
y, distance from the interface [cm].

Greek symbols
α, $\equiv \lambda/(\rho C_p)$, thermal diffusivity of the first phase [cm^2/s];
ρ, density of the first phase [g/cm^3];
ρ_d, density of the second phase [g/cm^3];
λ, effective thermal conductivity of the first phase [cal/sm°K];
θ, angle [rad];
τ, dummy integration variable;
ξ, dummy integration variable;
$\gamma(t)$, function defined in equation (38) or (44).

Subscripts
d, pertaining to the second phase;
h, pertaining to the heat-transfer process;
m, pertaining to the mass-transfer process;
sat, saturated (pertaining to equilibrium at the phase interphase);
w, at the interface, $r = R(t)$;
0, evaluated at $t = 0$;
∞, far from the spherical second phase center.

Superscripts
g, pertaining to two component second phase.

INTRODUCTION

EXACT treatment of a stationary spherical second phase growth as governed by simultaneous heat and mass transfer limitations has been recently demonstrated [1]. The second phase can be a bubble (gas), a droplet (liquid), or a particle (solid). However, quite frequently in many fields of applied science, the second phase is always moving, i.e. there exists a translatory motion of the center of the second phase relative to the surrounding first phase. The growth law for a moving second phase with very small density ratio ρ_d/ρ in the heat or mass transfer limited cases has been otained recently by Ruckenstein and Davis [2, 3]. The valid boundary layer approximation has been used by them. Also the growth law for a moving second phase with very large density ratio ρ_d/ρ in the heat or mass transfer limited cases has been obtained by Ruckenstein [4] and Chao [5]. They used combined stationary interface and boundary layer approximations. However, as pointed out in [1], in general, heat and mass transfer limitations occur at the same time, and the temperature of the growing second phase is not known *a priori*. Thus, one should view the heat or mass transfer limited cases as asymptotic extremes of the simultaneous heat and mass transfer limited cases, as will be demonstrated in the Appendices. As a first guess, of course, one would expect that somehow the parameters characterizing the velocity flow field of the first phase should appear in the compatibility condition from which the *a priori* unknown second phase temperature is calculated. Yet, it turns out to be the reverse, i.e. the velocity flow field of the first phase, or the fact that the second phase is moving, does not come into play at all in the boundary layer approximation (for small density ratio ρ_d/ρ) and in the combined stationary interface and boundary layer approximations (for large density ratio ρ_d/ρ) as far as calculating the *a priori* unknown second phase temperature concerned, as will be demonstrated later. In this work, only the high Reynolds number flow (or potential flow) case will be demonstrated. The low Reynolds number flow (or Stokes flow) case will be treated elsewhere [6].

STATEMENT OF THE PROBLEM

The problem under consideration is as follows: A spherical second phase of size, R_0, is produced in an environment, i.e. the first phase, at time $t = 0$. The second phase can be a bubble (gas), a droplet (liquid), or a particle (solid). At time $t = 0$, the entire second phase is assumed to have attained a certain proper equilibrium temperature T_w, i.e. the wet-bulb temperature, and remain at this temperature throughout the growth process. That is, one assumes that throughout the entire transient growth process a constant T_w exists, corresponding to a constant $c_{sat}(T_w)$ which must be found as part of the problem solution (see Discussion). At times $t > 0$, the second phase starts to grow due to both heat and mass transfer driving forces and move in the first phase. The center of the second phase is assumed to move at a velocity u_∞ relative to stationary coordinates and the flow field around the second phase is assumed to be approximated by the potential flow.

The second phase is characterized by the following parameters: initial radius, R_0, density, ρ_d, latent heat of phase transition, $L(T_w)$ (<0 for endothermic; >0 for exothermic), and saturation concentration $c_{sat}(T_w)$; the first phase is characterized by the following parameters: density, ρ, specific heat, c_p, effective thermoconductivity, λ, and effective Fick's diffusion coefficient, D. The first phase is initially at a uniform temperature T_∞ and concentration c_∞ ($> c_{sat}$), while the second phase is assumed to have uniform constant temperature T_w and concentration c_d throughout the growth process. During the growth process, i.e. $t \geqslant 0$, the system is described by the following equations,

$$\frac{DT}{Dt} = \alpha\nabla^2 T, \qquad R(t) \leqslant r \leqslant \infty \tag{1a}$$

$$\frac{Dc}{Dt} = D\nabla^2 c, \qquad R(t) \leqslant r \leqslant \infty \tag{1b}$$

with

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + (v_{rt} + v_{rg})\frac{\partial}{\partial n} + \frac{v_{\theta t}}{r}\frac{\partial}{\partial \theta}$$

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)$$

$$v_{rg} = \frac{R^2}{r^2}\cdot\left(1 - \frac{\rho_d}{\rho}\right)\cdot\dot{R}$$

$$v_{rt} = -u_\infty\cdot\left(1 - \frac{R^3}{r^3}\right)\cdot\cos\theta$$

$$v_{\theta t} = u_\infty\cdot\left(1 + \frac{R^3}{2r^3}\right)\cdot\sin\theta$$

$$T(r,\theta,0) = T_\infty \tag{2a}$$

$$c(r,\theta,0) = c_\infty \tag{2b}$$

$$T(\infty,\theta,t) = T_\infty \tag{3a}$$

$$c(\infty,\theta,t) = c_\infty \tag{3b}$$

$$T(R(t),\theta,t) = T_w \tag{4a}$$

$$c(R(t),\theta,t) = c_{sat}(T_w) \tag{4b}$$

$$\rho_d\dot{R} = \frac{\lambda}{-L(T_w)}\cdot\frac{1}{2}\cdot\int_0^\pi\left(\frac{\partial T}{\partial r}\right)_{r=R(t)}\sin\theta\,d\theta \tag{5a}$$

$$\rho_d\dot{R} = \frac{D\rho}{c_d - c_{sat}(T_w)}\cdot\frac{1}{2}\cdot\int_0^\pi\left(\frac{\partial c}{\partial r}\right)_{r=R(t)}\sin\theta\,d\theta$$

$$R(0) = R_0 \tag{6}$$

where $\alpha \equiv \lambda/(\rho c_p)$ is the thermal diffusivity of the first phase. The problem is to find the *a priori* unknown temperature T_w and obtain the growth law of the second phase, $R(t)$.

METHOD OF SOLUTION

The key to this physically important problem is to recognize that the growth laws obtained from either heat or mass transfer viewpoints must be identical. Thus, one obtains the compatibility condition from which T_w is calculated (see below). The exact solution of this very complicated problem is still yet to be found. However, for certain asymptotic extremes, various kinds of valid approximations are available.

(i) *Boundary layer approximation for the small density ratio ρ_d/ρ case*

With the small density ratio

$$1 \gg \rho_d/\rho \tag{7}$$

and the thin boundary-layer assumptions, i.e.

$$\frac{\partial^2 T}{\partial r^2} \gg \frac{2}{r}\frac{\partial T}{\partial r} \tag{8a}$$

$$\frac{\partial^2 c}{\partial r^2} \gg \frac{2}{r}\frac{\partial c}{\partial r} \tag{8b}$$

$$\frac{\partial^2 T}{\partial r^2} \gg \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right) \tag{8c}$$

$$\frac{\partial^2 c}{\partial r^2} \gg \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial c}{\partial\theta}\right) \tag{8d}$$

and

$$0 \leqslant \frac{y}{R(t)} \equiv \frac{r - R(t)}{R(t)} \ll 1, \tag{8e}$$

the governing equations (1)–(6) are simplified into the following form [2, 3],

$$\frac{\partial T}{\partial t} - y\left(3\cdot\frac{u_\infty}{R}\cdot\cos\theta + \frac{2}{R}\frac{dR}{dt}\right)\frac{\partial T}{\partial y} + \frac{3}{2}\cdot\frac{u_\infty}{R} \times \sin\theta\frac{\partial T}{\partial\theta} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{9a}$$

From the heat transfer viewpoint, i.e. equations (9a), (10a), (11a), (12a), (13a) and (14), the temperature variable $T(y, \theta, t)$ satisfies the same boundary value problem as in [2] and [3]. Thus, one gets

$$R_h(t) = R_0 - \frac{Na_h^g}{2}\cdot\sqrt{\left(\frac{\alpha}{\pi}\right)}\cdot G_h(t) \tag{15}$$

where

$$Na_h^g \equiv \frac{\rho}{\rho_d}\cdot B_h^g \equiv \frac{\rho}{\rho_d}\cdot\frac{c_p(T_\infty - T_w)}{L(T_w)} \tag{16}$$

and $G_h(t)$ is defined by $G(t)$

$$G(t) \equiv \int_0^t\int_0^\pi \frac{\sin\theta\, d\theta\, d\tau}{\left\{\int_0^\tau \exp\left[\int_\tau^p \left(6A(\xi)\cdot\frac{1 - (\tan^2\theta/2)\exp\left(3\int_\tau^\xi A(s)\,ds\right)}{1 + (\tan^2\theta/2)\exp\left(3\int_\tau^\xi A(s)\,ds\right)} + 4B(\xi)\right)d\xi\right] dp\right\}^{\frac{1}{2}}} \tag{17}$$

$$\frac{\partial c}{\partial t} - y\left(3\cdot\frac{u_\infty}{R}\cdot\cos\theta + \frac{2}{R}\frac{dR}{dt}\right)\frac{\partial c}{\partial y} + \frac{3}{2} \times \frac{u_\infty}{R}\cdot\sin\theta\frac{\partial c}{\partial\theta} = D\frac{\partial^2 c}{\partial y^2} \tag{9b}$$

$$T(y, \theta, 0) = T_\infty \tag{10a}$$

$$c(y, \theta, 0) = c_\infty \tag{10b}$$

$$T(\infty, \theta, t) = T_\infty \tag{11a}$$

$$c(\infty, \theta, t) = c_\infty \tag{11b}$$

$$T(0, \theta, t) = T_w \tag{12a}$$

$$c(0, \theta, t) = c_{sat}(T_w) \tag{12b}$$

$$\rho_d\dot{R} = \frac{\lambda}{-L(T_w)}\cdot\frac{1}{2}\cdot\int_0^\pi\left(\frac{\partial T}{\partial y}\right)_{y=0}\sin\theta\, d\theta \tag{13a}$$

$$\rho_d\dot{R} = \frac{D\rho}{c_d - c_{sat}(T_w)}\cdot\frac{1}{2}\cdot\int_0^\pi\left(\frac{\partial c}{\partial y}\right)_{y=0}\sin\theta\, d\theta \tag{13b}$$

$$R(0) = R_0. \tag{14}$$

with

$$A(t) = A_h(t) \equiv \frac{u_\infty(R_h(t))}{R_h(t)} \tag{18}$$

and

$$B(t) = B_h(t) \equiv \frac{d\ln R(t)}{dt}. \tag{19}$$

From the mass transfer viewpoint, i.e. equations (9b), (10b), (11b), (12b), (13b) and (14), the concentration variable $c(y, \theta, t)$ satisfies the same boundary value problem as in [2] and [3]. Thus, one gets

$$R_m(t) = R_0 - \frac{Na_m^g}{2}\cdot\sqrt{\left(\frac{D}{\pi}\right)}\cdot G_m(t) \tag{20}$$

where

$$Na_m^g \equiv \frac{\rho}{\rho_d}\cdot B_m^g \equiv \frac{\rho}{\rho_d}\cdot\frac{c_{sat}(T_w) - c_\infty}{c_d - c_{sat}(T_w)} \tag{21}$$

and $G_m(t)$ is given by $G(t)$ (equation (17)) with

$$A(t) = A_m(t) \equiv \frac{u_\infty(R_m(t))}{R_m(t)} \tag{22}$$

and

$$B(t) = B_m(t) \equiv \frac{\mathrm{d} \ln R_m(t)}{\mathrm{d}t}. \quad (23)$$

The uniqueness of the growth law of the second phase, i.e. $R_h(t) = R_m(t) \equiv R(t)$, gives the following compatbility condition:

$$Na_h^g \cdot \sqrt{\alpha} = Na_m^g \sqrt{D} \quad (24)$$

or

$$B_h^g \cdot \sqrt{\alpha} = B_m^g \sqrt{D}. \quad (25)$$

The value of T_w (hence $c_{sat}(T_w)$, B_h^g and B_m^g) must be properly chosen so that the compatibility condition equation (25) is satisfied.

Then, the required growth law is given by

$$R(t) = R_0 - \frac{Na_h^g}{2} \cdot \sqrt{\left(\frac{\alpha}{\pi}\right)} \cdot G(t) \quad (26a)$$

$$= R_0 - \frac{Na_m^g}{2} \cdot \sqrt{\left(\frac{D}{\pi}\right)} \cdot G(t) \quad (26b)$$

where $G(t)$ is given by equation (17) with

$$A(t) = \frac{u_\infty(R(t))}{R(t)} \quad (27)$$

and

$$B(t) = \frac{\mathrm{d} \ln R(t)}{\mathrm{d}t}. \quad (28)$$

The remarkable result is that, indeed, the parameters characterizing the flow field in the first phase does not appear in the compatibility condition equation (25) at all. In other words, the fact that the second phase is moving fast, does not come into play at all as far as calculating the *a priori* unknown second phase temperature T_w concerned. The compatibility condition equation (25) is completely identical to the compatibility condition when the second phase is stationary (cf. equation (A36) in [1]). Another remarkable result is the fact that the second phase initial size, R_0, does not appear in the compatibility condition equation (25) within the validity of the approximations. Few physically important asymptotic cases are considered in the Appendix A.

(ii) *Combined stationary interface and boundary layer approximations for the large density ratio ρ_d/ρ case*

With the stationary interface approximation, i.e. treating $R(t)$ as a constant while solving the diffusion equations, and the boundary layer approximation, i.e.

$$\frac{\partial^2 T}{\partial r^2} \gg \frac{2}{r}\frac{\partial T}{\partial r} \quad (29a)$$

$$\frac{\partial^2 c}{\partial r^2} \gg \frac{2}{r}\frac{\partial c}{\partial r} \quad (29b)$$

$$\frac{\partial^2 T}{\partial r^2} \gg \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial T}{\partial \theta}\right) \quad (29c)$$

$$\frac{\partial^2 c}{\partial r^2} \gg \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial c}{\partial \theta}\right) \quad (29d)$$

and

$$0 \leqslant \frac{y}{R} \equiv \frac{r - R(t)}{R(t)} \ll 1 \quad (29e)$$

the governing equations (1)–(6) are simplified into the following form [4, 5],

$$\frac{\partial T}{\partial t} - y \cdot 3 \cdot \frac{u_\infty}{R} \cdot \cos\theta \cdot \frac{\partial T}{\partial y} + \frac{3}{2}\frac{u_\infty}{R} \times \sin\theta \cdot \frac{\partial T}{\partial \theta} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (30a)$$

$$\frac{\partial c}{\partial t} - y \cdot 3 \cdot \frac{u_\infty}{R} \cdot \cos\theta \cdot \frac{\partial c}{\partial y} + \frac{3}{2} \cdot \frac{u_\infty}{R} \times \sin\theta \cdot \frac{\partial c}{\partial \theta} = D \frac{\partial^2 c}{\partial y^2} \quad (30b)$$

$$T(y, \theta, 0) = T_\infty \quad (31a)$$

$$c(y, \theta, 0) = c_\infty \quad (31b)$$

$$T(\infty, \theta, t) = T_\infty \quad (32a)$$

$$c(\infty, \theta, t) = c_\infty \quad (32b)$$

$$T(0, \theta, t) = T_w \quad (33a)$$

$$c(0, \theta, t) = c_{sat}(T_w) \quad (33b)$$

$$\rho_d \dot{R} = \frac{\lambda}{-L(T_w)} \cdot \frac{1}{2} \cdot \int_0^\pi \left(\frac{\partial T}{\partial y}\right)_{y=0} \cdot \sin\theta \, \mathrm{d}\theta \quad (34a)$$

$$\rho_d \dot{R} = \frac{D\rho}{c_d - c_{sat}(T_w)} \cdot \frac{1}{2} \cdot \int_0^\pi \left(\frac{\partial c}{\partial y}\right)_{y=0} \cdot \sin\theta \, d\theta \quad (34b)$$

$$R(0) = R_0. \quad (35)$$

From the heat-transfer viewpoint, i.e. equations (30a), (31a), (32a), (33a), (34a) and (35), the temperature variable $T(y, \theta, t)$ satisfies the same boundary value problem as in [4] and [5]. Thus, one gets

$$R_h(t) = R_0 - \sqrt{\left(\frac{2\alpha}{\pi}\right)} \cdot Na_h^g \cdot H_h(t) \quad (36)$$

where $H_h(t)$ is given by $H(t)$

$$H(t) \equiv \int_0^t \frac{\sqrt{[\gamma(\tau)]}}{\pi} \times \int_0^\pi \frac{\sin^3\theta \, d\theta \, d\tau}{\{[f(\tau,\theta) - \cos\theta] - \frac{1}{3}[f^3(\tau,\theta) - \cos^3\theta]\}^{\frac{1}{2}}} \quad (37a)$$

with

$$f(t,\theta) \equiv \frac{1 - \dfrac{1-\cos\theta}{1+\cos\theta} \cdot \exp[-\gamma(t) \cdot t]}{1 + \dfrac{1-\cos\theta}{1+\cos\theta} \cdot \exp[-\gamma(t) \cdot t]} \quad (37b)$$

and

$$\gamma(t) = \gamma_h(t) \equiv 3 \cdot \frac{u_\infty(R_h(t))}{R_h(t)}. \quad (38)$$

From the mass-transfer viewpoint, i.e. equations (30b), (31b), (32b), (33b), (34b) and (35), the concentration variable $c(y, \theta, t)$ satisfies the same boundary value problem as in [4] and [5]. Thus, one gets

$$R_m(t) = R_0 - \sqrt{\left(\frac{2D}{\pi}\right)} \cdot Na_m^g \cdot H_m(t) \quad (39)$$

where $H_m(t)$ is given by $H(t)$ (equation (37)) with

$$\gamma(t) = \gamma_m(t) \equiv 3 \cdot \frac{u_\infty(R_m(t))}{R_m(t)}. \quad (40)$$

The uniqueness of the growth law of the second phase, i.e. $R_h(t) = R_m(t) \equiv R(t)$, gives the compatibility condition as follows:

$$Na_h^g \cdot \sqrt{\alpha} = Na_m^g \cdot \sqrt{D} \quad (41)$$

or

$$B_h^g \cdot \sqrt{\alpha} = B_m^g \cdot \sqrt{D}. \quad (42)$$

The value of T_w (hence $c_{sat}(T_w)$, B_h^g and B_m^g) must be properly chosen so that the compatibility condition equation (42) is satisfied. Then, the required growth law is given by

$$R(t) = R_0 - \sqrt{\left(\frac{2\alpha}{\pi}\right)} \cdot Na_h^g \cdot H(t) \quad (43a)$$

$$= R_0 - \sqrt{\left(\frac{2D}{\pi}\right)} \cdot Na_m^g \cdot H(t) \quad (43b)$$

where $H(t)$ is given by equation (37) with

$$\gamma(t) = 3 \cdot \frac{u_\infty(R(t))}{R(t)}. \quad (44)$$

Again, both the parameters characterizing the flow field in the first phase and the initial second phase sphere radius do not appear in the compatibility condition equation (42). In fact, equations (25) and (42) are identical. Few physically important asymptotic cases are considered in the Appendix B.

DISCUSSION

It is assumed that all the physical and transport properties of the second and first phases are constant and there exists a local equilibrium relationship $c_{sat}(T_w)$ at $r = R(t)$ throughout the growth process. The compatibility condition equation (25) (for small density ratio (ρ_d/ρ) or equation (42) (for large density ratio ρ_d/ρ) is a necessary and sufficient condition for the existence of the stated constant interface condition solution, i.e. it guarantees the uniqueness of the growth law, $R(t)$. Thus, the basic assumption of strictly constant T_w (and, thus, constant $c_{sat}(T_w)$) is automatically justified *a posteriori* for second phase growth problems of the type considered here. Physically, the necessary and sufficient compatibility condition means that the second phase can grow if one maintains $T(\infty, t) = T_\infty$ and $c(\infty, t) = c_\infty$ throughout the growth process.

CONCLUSIONS

Two valid approximate treatments of the growth of a fast moving spherical second phase in the presence of simultaneous heat and mass-transfer limitations have been demonstrated. In general, a trial-and-error method must first be used to solve the compatibility condition, equation (25) or (42). Having thus determined T_w, the growth law is then readily obtained. Formally, we have demonstrated in the Appendices that heat- or mass-transfer limited cases can correspond to two different asymptotic cases of simultaneous heat- and mass-transfer limited cases. These results should be useful in several physical sciences.

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APPENDIX A

Within the validity of the boundary layer approximation, two asymptotic cases according to the Lewis number $Le \equiv D/\alpha$ are considered as follows:

(a) $Le \gg 1,$ (A.1)

the compatibility condition equation (25) gives

$$|B_m^g| \ll 1. \tag{A.2}$$

Then, equation (21) gives

$$c_{sat}(T_w) \approx c_\infty. \tag{A.3}$$

This determines T_w and then the required growth law is obtained from equations (15)–(19). This is the growth law for the second phase in the heat transfer limited case with small density ratio ρ_d/ρ. Formally, this result is equivalent to the case when $c_d \approx c_\infty$ (and, thus, $c_{sat}(T_w) \approx c_\infty$) [3].

(b) $Le \ll 1,$ (A.4)

the compatibility condition equation (25) gives

$$|B_h^g| \ll 1. \tag{A.5}$$

Then, equation (16) gives

$$T_w \approx T_\infty. \tag{A.6}$$

Then, the required growth law is obtained from equations (20)–(23). This is the growth law for the second phase in the mass transfer limited case with small density ratio ρ_d/ρ. Formally, this result is equivalent to the case when $L(T_w) \approx 0$ (and, thus, $T_w \approx T_\infty$) [2].

APPENDIX B

Within the validity of the combined stationary interface and boundary layer approximations, two asymptotic cases according to the Lewis number $Le \equiv D/\alpha$ are considered as follows:

(a) $Le \gg 1,$ (B.1)

the compatibility condition equation (42) gives

$$|B_m^g| \ll 1. \tag{B.2}$$

Then, equation (21) gives

$$c_{sat}(T_w) \approx c_\infty. \tag{B.3}$$

This determines T_w and then the required growth law is obtained from equations (36)–(38). This is the growth law for the second phase in the heat transfer limited case with large density ratio ρ_d/ρ. Formally, this result is equivalent to the case when $c_d \approx c_\infty$ (and, thus, $c_{sat}(T_w) \approx c_\infty$) [4, 5].

(b) $Le \ll 1,$ (B.4)

the compatibility condition equation (42) gives

$$|B_h^g| \ll 1. \quad \text{(B.5)}$$

Then, equation (21) gives

$$T_w \approx T_\infty. \quad \text{(B.6)}$$

Then, the required growth law is obtained from equations (39) and (40). This is the growth law for the second phase in the mass transfer limited case with small density ratio ρ_d/ρ. Formally, this result is equivalent to the case when $L(T_w) \approx 0$ (and, thus, $T_w \approx T_\infty$) [4, 5].

LOI DE CROISSANCE D'UNE PHASE SECONDAIRE SPHERIQUE EN DEPLACEMENT RAPIDE CONDITIONNEE PAR UN TRANSFERT SIMULTANE DE CHALEUR ET DE MASSE

Résumé—On a considéré un traitement théorique nouveau de la croissance d'une phase secondaire sphérique en déplacement rapide gouvernée par un transfert simultané de chaleur et de masse. La nouvelle méthode démontre que la solution de ces cas complexes couplés peut être reliée aux cas utilisables non couplés. Ainsi, on montre que les cas limites du transfert de chaleur ou de masse sont les cas asymptotiques des cas limites du transfert simultané de chaleur et de masse.

WACHSTUMSGESETZ EINER SCHNELL BEWEGTEN, KUGELFÖRMIGEN ZWEITEN PHASE AUFGRUND DER EINSCHRÄNKUNGEN DURCH GLEICHZEITIGE WÄRME- UND STOFFÜBERTRAGUNG

Zusammenfassung—Es wird eine neue Behandlung angegeben für das Wachstumsgesetz einer schnell bewegten, kugelförmigen zweiten Phase aufgrund der Einschränkungen durch gleichzeitige Wärme- und Stoffübertragung.

Die neue Methode zeigt, dass die Lösung dieses komplex gekoppelten Falles auf verfügbare nicht gekoppelte Fälle bezogen werden kann. Reine Wärme- oder Stoffübertragung erweisen sich damit als asymptotische Fälle der Vorgänge bei gleichzeitiger Wärme- und Stoffübertragung.

ЗАКОН РОСТА БЫСТРО ПЕРЕМЕЩАЮЩЕЙСЯ СФЕРИЧЕСКОЙ ВТОРОЙ ФАЗЫ ПРИ ОДНОВРЕМЕННОМ ДЕЙСТВИИ ОГРАНИЧЕНИЙ ТЕПЛО-И МАССОПЕРЕНОСА

Аннотация—Рассматривается новый метод теоретического анализа роста быстро перемещающейся сферической второй фазы при одновременном действии ограничений тепло-и массопереноса. Этот метод показывает, что решение взаимозависимых процессов тепло-и массообмена можно свести к решениям независимых процессов теплообмена и массообмена. Таким образом показано, что случай, ограниченный либо переносом тепла, либо переносом массы, является асимптотическии вариантом случаев, ограниченных одновременным переносом тепла и массы.